Extended Essay

Physics

Title: How does the increased precision of the Hubble constant gives us a better understanding of the age of the universe?

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1.1 INTRODUCTION

During decades the Hubble law has increased our knowledge about the age of the universe. Hubble's law measures the velocity of relative objects observed in space. The law is expressed by the equation of $v = H_0 D$. Where H_0 is the Hubble constant which is proportional to the distance D to a galaxy, and v stands for recessional velocity. H_0 belongs to the value of H which is a time dependant value, often used for the Hubble constant at the current time.[1] As at 2012, the Hubble constant gives an expansion rate of the universe of 69.32±0.80 (km/s)/Mpc. These measurements can be interpreted to show that a galaxy from a distance of 1 Mpc has a recessional velocity of 69.32±0.80 km/s. This observations were measured by the probe originally known as MAP, which stands for Microwave Anisotropy Probe, throughout its 9 year mission from 2001 to 2010. Later, in 2002, the probe was renamed to WMAP.[2] The WMAP was a spacecraft that measured the temperature differences in the cosmic microwave background. Also known as the CMB, this was the leftover radiation heat from the Big Bang. In 2012 the WMAP released an estimate of the age of the universe, $(13.772 \pm 0.059) \times 10^9$ or 13.772 billion years, with an uncertainty of 59 million years. Going back in time, this can be related to the Cosmic Background Explorer, or COBE. Which was an Explorer class satellite that was launched on November the 18th, 1989 and ended four years later on December 23, 1993. One of COBE's main objectives was to measure and investigate the cosmic microwave background radiation (CMB)[3]. This was for allowing the scientific community to have a better understanding of the universe. The mission consisted of taking accurate measurements inside the entire celestial sphere of the diffuse radiation between the range of 1 micrometer and 1 centimeter. The 2,270 kg explorer satellite carried three fundamental instruments. A FIRAS, Far Infrared Absolute Spectrophotometer, which had the role of comparing the spectrum of the cosmic microwave background radiation with a blackbody. Secondly, a DMR, Differential Microwave Radiometer,

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[1] "Hubble's Law." *Wikipedia*, Wikimedia Foundation, 28 July 2019, en.wikipedia.org/wiki/Hubble's_law.

[2] "Wilkinson Microwave Anisotropy Probe." *Wikipedia*, Wikimedia Foundation, 14 July 2019, en.wikipedia.org/wiki/Wilkinson_Microwave_Anisotropy_Probe.

[3] *NASA*, NASA, science.nasa.gov/missions/cobe.

with the purpose of mapping out the cosmic radiation. Finally a DIRBE, Diffuse Infrared Background Experiment, which served to search for the cosmic infrared background radiation.

1.1.1. First Observations

The very first hints that the universe has a finite age and was not static, originated from the observations of recession velocities by Vesto Slipher, in addition with the distances to the galaxies by Edwin Hubble. During the 1920's, Hubble observed different spectra taken from Cepheid stars further away galaxies to measure their redshift. The redshift in their wavelength indicated by the Doppler effect, showed that the distant stars where moving away. He used the Cepheid variable stars as a standard candle to measure the distance to the galaxy observed[4]. A standard candle can be a supernovae or a variable star which has a known luminosity, therefore it is used as a standard[5]. On the other side, Cepheid variable stars pulsate through different luminosities in a predictable form. Through these methods is how Hubble calculated the age of the universe giving a value for the life-span of the cosmos.[6] However, nowadays we know that the value of the age of the universe calculated by Hubble is incorrect.

1.1.2. Doppler`s Red Shift

The Hubble law talks about the direct correlation between the distance to a galaxy and moving velocity, which is determined by the red shift[7]. The light from distant stars, from the observed galaxy, give away spectral characteristics of the atoms in the gases around them. This spectra can appear to be shifted towards the red end of the spectrum.[8] This redshift indicates that the light observed has a larger wavelength and the greater the red in the spectrum, the faster the galaxy is

[4] https://lco.global/spacebook/cepheid-variable-stars-supernovae-and-distance-measurement/

[5] *Standard Candles in Astronomy*, universe-review.ca/R02-07-candle.htm.

[6] "Age of the Universe." *Wikipedia*, Wikimedia Foundation, 20 July 2019, en.wikipedia.org/wiki/Age_of_the_universe.

[7] Harland, David M. "Cosmic Background Explorer." *Encyclopædia Britannica*, Encyclopædia Britannica, Inc., www.britannica.com/topic/Cosmic-Background-Explorer.

[8] "Red Shift." *Doppler Redshift*, hyperphysics.phy-astr.gsu.edu/hbase/Astro/redshf.html#c1.

moving away from us. This is the astronomical version of the Doppler effect. The Doppler effect is the change in frequency or wavelength that undertakes a wave in contrast to an observer[9]. Therefore we can say that Hubble's law is one of the most important arguments that support the Big Bang because it supports the theory of cosmic inflation which is the exponential expansion of space at the early stages of the cosmos.

2.1 ANALYSIS AND DISCUSSION

Below are some examples of how the Hubble constant changes over time. With the purpose of simplification, data was taken from four years in between, when possible.

Table 1. Hubble constant values over time.

Date	Hubble constant (km/s) / Mpc	Observer
28/03/2019	74.03 ± 1.42	Hubble Space Telescope
February 2015	67.74 ± 0.46	Planck Mission
20/12/2012	69.32 ± 0.80	WMAP
2007	$70.4^{+1.5}_{-1.6}$	WMAP
August 2006	$76.9\pm^{3.9}_{3.4}\pm^{10.0}_{8.0}$	Chandra X-ray Observatory
May 2001	72 ± 8	Hubble Space Telescope Key Project
Early's 1970	~55	Allan Sandage and Gustav Tammann
May 1958	75	Allan Sandage
1931	625	Georges Lemaitre

[9] "Doppler Effect." *Wikipedia*, Wikimedia Foundation, 28 July 2019, en.wikipedia.org/wiki/Doppler_effect.

1929	500	Edwin Hubble, Hooker Telescope

Edwin Hubble, Hooker Telescope

In 1929, Hubble stated that there is a linear correlation between distances and velocities from an observed nebulae. Therefore distances can be calculated from the velocities. In order to calculate distances on a larger scale, Mr. Humason initiated a program at Mount Wilson which determined the velocities of the most distant nebulae observed. The observed bright nebulae was N.G.C. 7619, had a velocity of $3910 km/sec$. Then with the K constant equal to 500, an old term used as the Hubble Constant, the distance of the nebulae was found. It was at a distance of 7.8×10^8 parsecs[10].The following table shows the 24 different Nebulae distances estimated from stars involved or from mean luminosities in a cluster. Where r = distance in units of 10^8 parsecs, and where v = the measured velocities in km/sec.

Table 2. Nebulae distances estimated from cluster[11]

Object	Distance r *units of* 10^8 *parsecs*	Velocity v $km/sec.$
S. Mag.	0.032	+170
L. Mag.	0.034	+290
N.G.C. 6822	0.214	-130
598	0.263	-70
221	0.275	-185
224	0.275	-220
5457	0.45	+200
4736	0.5	+290

[10] Hubble, Edwin. “A Relation between Distance and Radial Velocity among Extra-Galactic Nebulae.” *PNAS*, 1929, www.pnas.org/content/pnas/15/3/168.full.pdf.
[11] Ibid.

5194	0.5	+270
4449	0.63	+200
4214	0.8	+300
3031	0.9	-30
3627	0.9	+650
4826	0.9	+150
5236	0.9	+500
1068	1.0	+920
5055	1.1	+450
7331	1.1	+500
4258	1.4	+500
4151	1.7	+960
4382	2.0	+500
4472	2.0	+850
4486	2.0	+800
4649	2.0	+1090

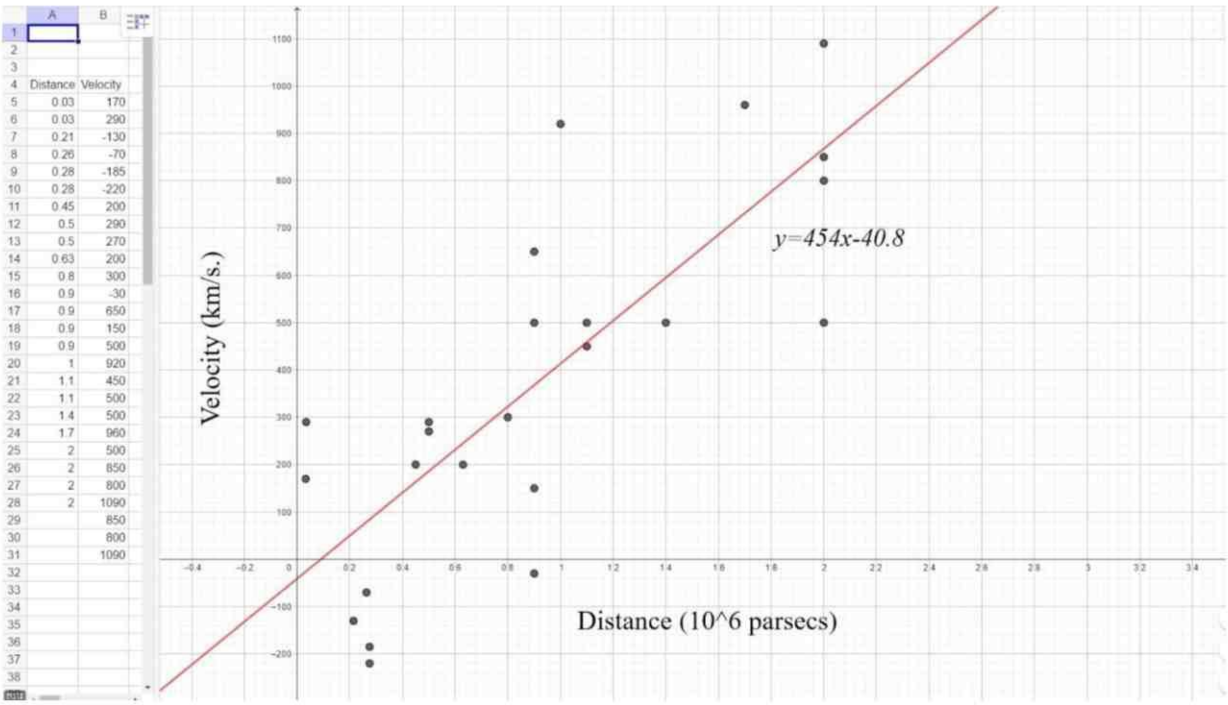
According to Edwin Hubble, the data shown in Table 4, only the first 7 distances are the most reliable. However the rest thirteen distances are not considered to be as reliable as there are probable error in the measurements due to the criterion of stellar luminosity[12]. Nevertheless, these were considered to be the most valid values from that moment in time. The last four objects belong to the Virgo Cluster. The Virgo Cluster was at an estimated distance of 2×10^8 parsecs, which according to Harvard estimate, is equivalent to an estimate of ten million light years[13].

[12] Hubble, Edwin. "A Relation between Distance and Radial Velocity among Extra-Galactic Nebulae." *PNAS*, 1929, www.pnas.org/content/pnas/15/3/168.full.pdf
[13] Ibid.

Nowadays, with improved methodology we know that the Virgo Cluster is at 65 million light years away from Earth.

The following figure shows a graph of the velocity-distance relation from the data collected among the extra-galactic nebulae. The velocity and distance values of the nebulae were taken from the Table 4 and plotted. For plotting the data collected, the GeoGebra Classic software was used.

Figure 1. Velocity-Distance relation



Distance	Velocity
0.03	170
0.03	290
0.21	-130
0.26	-70
0.28	-185
0.28	-220
0.45	200
0.5	290
0.5	270
0.63	200
0.8	300
0.9	-30
0.9	650
0.9	150
0.9	500
1	920
1.1	450
1.1	500
1.4	500
1.7	960
2	500
2	850
2	800
2	1090
	850
	800
	1090

The radial velocities are plotted on the *x-axis* and the distances, estimated from the involved stars and mean luminosities of nebulae in a cluster, on the *y-axis.* The line was made through linear regression by the GeoGebra program. By using a linear regression model, an estimation for the Hubble constant can be found by $y = 454x - 40.8$. In this equation, 454 is the constant, which is similar to the value calculated by Edwin Hubble. This scatter graph shows that distances of individual nebulae can be calculated from velocities[14]. As seen in Figure 1, the data in the

[14] Hubble, Edwin. "A Relation between Distance and Radial Velocity among Extra-Galactic Nebulae." *PNAS*, 1929, www.pnas.org/content/pnas/15/3/168.full.pdf

table form a linear correlation between distance and velocity. Hubble states that as velocities are assumed to vary with the distances, the K term represents the velocity given by this effect. By means of the following equation, the results in the table below were obtained. $rK + X \cos\alpha \cos\delta + Y \sin\alpha \cos\delta + Z \sin\delta = v$. [15]

Table 3. Results[16]

Term	24 Objects	9 Groups
X	-65 ± 50	$+3 \pm 70$
Y	$+226 \pm 95$	$+230 \pm 120$
Z	-195 ± 40	-133 ± 70
K	$+465 \pm 50$	$+513 \pm 60$ *km./sec. per* 10^4 *parsecs.*
A	286º	269º
D	+40º	+33º
V_0	306 km./sec.	247 km./sec.

With these results we can now calculate the age of the universe, as at 1929 constant estimates. By following the same procedures as in Table 4, the calculations give an estimate for the age of the universe of 1.955 billion years. The calculations are the following:

$$\frac{1}{500 \times 3.2408 \times 10^{-20}} = 6.171 \times 10^{16}$$

$$= 6.171 \times \frac{10^{16}}{3600 \times 24 \times 365.26}$$

$$= 1.955 \textit{ billion years}$$

[15] Hubble, Edwin. "A Relation between Distance and Radial Velocity among Extra-Galactic Nebulae." *PNAS*, 1929, www.pnas.org/content/pnas/15/3/168.full.pdf

[16] Ibid.

2.1.1 Calculating age of the universe

In the following table, an example for the procedure of calculating the age of the universe can be seen. For this example, the Hubble Constant chosen is of $H_0 = 74.03$, therefore the calculations estimate the age of the universe as in 2019, for this particular constant. This process will be repeated with the other constants.

Table 4. Example for calculating age of universe[17]

$$\left[\frac{1}{H_0} = T\right]$$

Units:

- 1 Mpc $= 10^6$ parsec
- 1 pc $= 206,265\,AU$
- AU 149,597,871 km.

Dimensionless constant $= \frac{1km.}{206265 \times 10^6 AU} = 3.2408 \times 10^{-20}$

$= \frac{1}{H_0 \times 3.2408 \times 10^{-20}} sec.$

For example, when $H_0 = 74.03$

$= \frac{1}{74.03 \times 3.2408 \times 10^{-20}}\ sec.$

$= 4.168 \times 10^{17} sec.$

$= 4.168 \frac{10^{17}}{3600 \times 24 \times 365.26}\ years$

$= 1.321 \times 10^{10} years$

$= 13.21\ billion\ years$

[17] "How Do You Calculate the Age of the Universe Using the Hubble Constant?: Socratic."

2.1.2. Hubble Space Telescope

On the article published on March 28, 2019, "*Large Magellanic Cloud Cepheid Standards Provide a 1% Foundation for the Determination of the Hubble Constant and Stronger Evidence for Physics Beyond ΛCDM*", (Adam G. Riess, Stefano Casertano, Wenlong Yuan, Lucas M. Macri and Dan Scolnic, 2019) it is presented an improved determination of the Hubble constant observed by the *Hubble Space Telescope (HST)*. The observations were made from a 70 long-period of Cepheids in the Large Magellanic Cloud *(LMC)*. The Cepheids taken into the observation were from the extragalactic Type Ia supernovae. This last one belongs to the Large Magellanic Cloud which is a satellite galaxy of the Milky Way which is nearly 200,000 light years from Earth[18]. The observations were made by the WFC3 photometric system which is an imager with powerful imaging capabilities mounted on the *HST*[19]. The first Hubble constant was found by using only the *Large Magellanic Cloud* DEB Cepheids. These Detached Eclipsing Binaries, provide a one-step distance determination to nearby galaxies[20]. By these terms a Hubble constant of $H_0 = 74.22 \pm 1.82\ km\ s^{-1} Mpc^{-1}$ was found. However by combining the *LMC* DEBs, masers, a microwave laser, in NGC 4258 and Milky Way parallaxes yields, gives a better estimate of $H_0 = 74.03 \pm 1.42\ km\ s^{-1} Mpc^{-1}$. These results include the systematic uncertainty of 1.91%[21].

The Hubble Space Telescope observed a best estimate of the Hubble Constant of $H_0 = 74.03$ in 2019. Following the procedures in Table 2, we can estimate the age of the universe to be of 13.21 billion years old.

$$= \frac{1}{74.03 \times 3.2408 \times 10^{-20}}\ sec.$$

$$= 4.168 \times 10^{17} sec.$$

$$= 4.168 \frac{10^{17}}{3600 \times 24 \times 365.26}\ years$$

[18] Administrator, NASA Content. "Large Magellanic Cloud." *NASA*, NASA, 9 Apr. 2015
[19] "Wide Field Camera 3." *STScI*
[20] https://iopscience.iop.org/article/10.1086/322378/fulltext/53801.text.html
[21] https://arxiv.org/pdf/1903.07603.pdf

$= 1.321 \times 10^{10} years$

$= 13.21\ billion\ years$

2.1.3. WMAP 2007

In 2007, "*Wilkinson Microwave Anisotropy Probe (WMAP) Three Year Observations: Implications for Cosmology*" (D. N. Sperge, R. Bean, O. Doré, M. R. Nolta, C. L. Bennett, J. Dunkley, G. Hinshaw, N. Jarosik, E. Komatsu, L. Page, H. V. Peiris, L. Verde, M. Halpern, R. S. Hill, A. Kogut, M. Limon, S. S. Meyer, N. Odegard, G. S. Tucker, J. L. Weiland, E. Wollack, E. L. Wright, 2007) presents three year WMAP temperature and polarization data. Throughout these observations, a Hubble constant of $H_0 = 70.4^{+1.5}_{-1.6} km\ s^{-1} Mpc^{-1}$ was found. This best estimate is the product of the mean of all the three years of observations. From the WMAP Lambda Cold Dark Matter (ΛCDM) data, a best fit value for the age of the universe can be estimated. This estimate gives, $t_0 = 13.73\ ^{+0.16}_{-0.15}$ galactic years (Gyr)[22].

2.1.4. Chandra X-ray Observatory

On "*Determination of the Cosmic Distance Scale from Sunyaev-Zel'dovich Effect and Chandra X-ray Measurements of High Redshift Galaxy Clusters*" (Massimiliano Bonamente, Marshall K. Joy, Samuel J. LaRoque, John E. Carlstrom, Erik D. Reese and Kyle S. Dawson, 2006) a Hubble constant of $H_0 = 76.9 \pm^{3.9}_{3.4} \pm^{10.0}_{8.0} km\ s^{-1} Mpc^{-1}$ is estimated. These estimations were obtained from the analysis of 38 clusters galaxies with the Chandra X-ray imaging spectroscopy and *OVRO-BIMA SZE* data, giving to the largest sample of data in 2006 to measure the Hubble constant. The *OVRO-BIMA SZE* was an experiment which "uses the signatures which galaxy clusters leave in the cosmic microwave background to measure the age and geometry of the universe"[23], (Massimiliano Bonamente, Marshall K. Joy, Samuel J. LaRoque, John E. Carlstrom, Erik D. Reese and Kyle S. Dawson, 2006). In order to get to the previous results, a hydrostatic equilibrium model was applied. We can say that a star is in hydrostatic equilibrium when the

[22] https://arxiv.org/pdf/astro-ph/0603449.pdf
[23] http://astro.uchicago.edu/sze/

gravitational force and the gas pressure is in balance[24]. This model was applied for radial variations in the cluster temperature. In addition, this model was used for sharp density gradients caused by the cooling of plasma in the cluster core[25], (Massimiliano Bonamente, Marshall K. Joy, Samuel J. LaRoque, John E. Carlstrom, Erik D. Reese and Kyle S. Dawson, 2006).

As at the year 2006, with Chandra X-ray Measurements we can estimate the age of the universe. With a Hubble Constant of $H_0 = 76.9$, we can estimate the universe to be 12.72 billion years old.

$$\frac{1}{76.9 \times 3.2408 \times 10^{-20}} = 4.013 \times 10^{17}$$
$$= 4.013 \times \frac{10^{17}}{3600 \times 24 \times 365.26}$$
$$= 12.72 \text{ billion years}$$

2.1.5. Hubble Space Telescope Key Project

The *Hubble Space Telescope Key Project* measured different Cepheid stars over a range of 60 to 400 Mpc. Based on different Cepheid distances, the following Hubble Constants were found. For the type Ia supernovae, a Hubble Constant of $H_0 = 71 \pm 2_r \pm 6_s$ was found, where 2_r is the random error, and 6_s, the systematic errors. For the Tully-Fisher relation, $H_0 = 71 \pm 3_r \pm 7_s$. For the surface brightness fluctuations the $H_0 = 70 \pm 5_r \pm 6_s$ was estimated. For the type II supernovae, $H_0 = 72 \pm 9_r \pm 7_s$, and for the fundamental plane, $H_0 = 82 \pm 6_r \pm 9_s$. When combining these results, the most consistent Hubble Constant estimate was of $H_0 = 72 \pm 8 \ km\ s^{-1} Mpc^{-1}$, (Wendy L. Freedman, Barry F. Madore and Brad K. Gibson, 2000).

With this result, we can now calculate an estimate for the age of the universe. By using the Hubble Constant found by the *Hubble Space Telescope Key Project* in the year 2000, the age of the universe is of 13.58 billion years old.

$$\frac{1}{72 \times 3.2408 \times 10^{-20}} = 4.286 \times 10^{17}$$
$$= 4.286 \times \frac{10^{17}}{3600 \times 24 \times 365.26}$$

[24] "Hydrostatic Equilibrium: COSMOS." *Centre for Astrophysics and Supercomputing*
[25] https://arxiv.org/pdf/astro-ph/0512349.pdf

$= 13.58\ billion\ years$

2.1.6. Allan Sandage

In May 1958, Allan Sandage stated that "the velocity-distance relation is the most important single instance where cosmological theory meets the test of observational astronomy", (Allan Sandage, 1958). He estimated the Hubble constant to be of 75 (km/s) / Mpc, result from the observations. These observations must be able to provide a numerical value for the expansion rate of the universe and change of the rate with time. For this distance-velocity observation, Sandage needed two numbers. These are H and $R_0 / R_0 H^2$, the deceleration parameter. He states that the relation between velocity and distance appears to be linear at any given cosmic time. This relation is expressed as $\frac{c\Delta\lambda}{\lambda} = Hr$. Where H is the Hubble constant. For achieving his results, Sandage utilizes for his observations, values from cepheid variables. These where cepheids of Eggen type A, B and C.

With Allan Sandage observations, we can calculate an estimate of the age of the universe. With the most reliable data in 1958, a Hubble Constant of 75 (km/s) / Mpc, the universe is expected to be of 13.04 billion years old.

$$\frac{1}{75\times 3.2408\times 10^{-20}} = 4.114\times 10^{17}$$
$$= 4.114\times \frac{10^{17}}{3600\times 24\times 365.26}$$
$$= 13.04\ billion\ years$$

2.1.7. Georges Lemaitre

Abbé G. Lemaitre in 1931 suggested that the radius of the universe is not constant as it is supposed to, but instead it varies with time. Lemaitre came to the following conclusion by combining the advantages of Einstein and de Sitter solutions. His conclusions were the following: First, that the radius of the universe increases without limit from an approached value, R_0 when $t = -\infty$. Second, he came to the conclusion that the receding velocities of extragalactic nebulae observed by Einstein are a cosmical effect of the expansion of the universe. The initial radius of the universe, R_0, can be calculated by the following formula. $R_0 = \frac{rc}{v\sqrt{3}}$. Lemaitre stated that "the period of light emitted under the same physical conditions has the same value everywhere when reckoned in proper time."[26]. With that being said, Lemaitre explained that this is the Doppler effect made by the variation of the radius of the universe. Therefore, supporting his idea that the expansion of the universe is not constant. The calculations give the following results.

Table 5. Lemaitre's Results[27]

$\frac{R'}{R} = 0.68 \times 10^{-27} cm.^{-1}$
$y = \frac{R_0}{R}$
$y = 0.0465$
$R_0 = 8.5 \times 10^{26} cm. = 2.7 \times 10^{8} parsecs.$
Therefore, $R_0 = 9 \times 10^{8} light\ years.$
$H_0 = 625$

As seen on the table above, the radius of the universe, given by R_0, is of 9×10^8 light-years. In addition, Lemaitre found a value for the expansion constant of $H_0 = 625$[28]. With this information we can estimate the age of the universe. As in 1931, the estimate for the age of the universe, using the $H_0 = 625$, was of 1.564 billion years.

$$\frac{1}{625 \times 3.2408 \times 10^{-20}} = 4.937 \times 10^{16}$$
$$= 4.937 \times \frac{10^{16}}{3600 \times 24 \times 365.26}$$
$$= 1.564\ billion\ years$$

[26] Lemaitre, Abbé G. "A Homogeneous Universe of Constant Mass and Increasing Radius accounting for the Radial Velocity of Extra-galactic Nebulœ." 1931
[27] Ibid.
[28] Helge Kragh. "Hubble Law or Hubble-Lemaître Law? The IAU Resolution", 2018

3. CONCLUSION

In conclusion, we can now say that the increased precision of the Hubble constant gives us a better understanding of the age of the universe. As seen through the investigation above, as the Hubble Constant increased it's precision, we've got closer to the most accurate estimates of the age of the universe today, which is to be 13.77 billion years. In the following table, the estimates for the age of the universe obtained from different Hubble Constants can be observed.

Table 6. Estimates for the age of the universe

Date	Project	Hubble Constant *(km./s) / Mpc*	Age of the universe *billion years*
March 28, 2018.	Hubble Space Telescope	$H_0 = 74.03 \pm 1.42$	13.21
2007	WMAP	$H_0 = 70.4^{+1.5}_{-1.6}$	13.73
2006	Chandra X-ray Observatory	$H_0 = 76.9 \pm^{3.9}_{3.4} \pm^{10.0}_{8.0}$	12.72
2000	Hubble Space Telescope Key Project	$H_0 = 72 \pm 8$	13.58
May 1958	Allan Sandage	$H_0 = 75$	13.04
1931	Georges Lemaitre	$H_0 = 625$	1.564
1929	Edwin Hubble, Hooker Telescope	$H_0 = 500$	1.955

As seen in Table 6, as the estimates for the Hubble Constant improve in precision, the estimate for the age of the universe does in the same way. In 1929, Edwin Hubble calculated a constant which related velocities and distances of extra-galactic nebulae that equal to 500. By the use of this constant, the age of the universe will be of 1.955 billion years old, more than ten billion years old that today's best estimate.The first most accurate estimate for the age of the universe was made by Allan Sandage in 1958. He calculated a Hubble Constant of $H_0 = 75$, which made an estimate for the age of the universe of 13.04 billion years old. However the most precise estimate for the age of the universe, from the chosen observers, was made by the WMAP probe in 2007. Throughout observations a constant of $H_0 = 70.4^{+1.5}_{-1.6}$ was found. This gives an estimate for the age of the universe of 13.73 billion years old.

Therefore with this evidence we can say that the increased precision of the Hubble constant gives us a better understanding of the age of the universe.

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